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ABSTRACT

Formulas for the scattering from an inhomogeneous sphere in fluid-saturated porous medium are used to construct a self-consistent effective medium approximation for the coefficients in Biot's equations of poroelasticity when the material constituting the porous solid frame is not homogeneous on the microscopic scale. The discussion is restricted to porous materials exhibiting both macroscopic and microscopic isotropy. Brown and Korringa have previously found the general form of these coefficients. The present results give explicit estimates of all the coefficients in terms of the moduli of the solid constituents. The results are also shown to be completely consistent with the well-known results of Gassmann and of Biot and Willis, as well as those of Brown and Korringa.

I. Introduction

Although Biot's theory of poroelasticity[1–3] has been quite successful in explaining laboratory data on man-made porous materials such as sintered packings of glass beads,[4–8] the theory has not been as successful at explaining data over a wide range of frequencies for naturally occurring materials such as porous sandstones[9] and granites[10].The reason for this failure of the theory is not well understood at present. However, since there are several clearly unrealistic assumptions made in the usual application of Biot's equations to earth materials, progress on the application of the theory may follow if some of these assumptions are replaced by more realistic ones.

The issue which will be addressed in the present work is the form of the coefficients in Biot's equations when the porous medium is composed not of a single granular material as is usually assumed but of several species of solid grains or of solid grains mixed together with clays and cement. The general form of these coefficients has already been studied by Brown and Korringa[11] and elucidated further by Korringa.[12] Their main result may be expressed as

$$H - \frac{4}{3}\mu = K + \sigma C, \tag{1}$$

$$C = \sigma \Big/ \Big[\frac{\sigma}{K_s} + \phi\Big(\frac{1}{K_f} - \frac{1}{K_\phi}\Big)\Big], \tag{2}$$

$$M = C/\sigma, \tag{3}$$

where

$$\sigma = 1 - K/K_s. \tag{4}$$

The constants $H, C,$ and M are coefficients in Biot's equations whose significance will become apparent in the next section. The other constants appearing in Eqs.(1)-(4) are the porosity ϕ and shear modulus μ of the porous frame, the bulk modulus of the pore fluid K_f, and three other bulk moduli characteristic of the porous frame:

$$\frac{1}{K} = -\frac{1}{V}\left(\frac{\partial V}{\partial p_d}\right)_{p_f}, \tag{5}$$

$$\frac{1}{K_s} = -\frac{1}{V}\left(\frac{\partial V}{\partial p_f}\right)_{p_d}, \tag{6}$$

and

$$\frac{1}{K_\phi} = -\frac{1}{V_\phi}\left(\frac{\partial V_\phi}{\partial p_f}\right)_{p_d} \tag{7}$$

where V is the total volume, $V_\phi = \phi V$ is the pore volume, p is the external pressure, p_f is the pore pressure, and $p_d = p - p_f$ is the differential pressure. Brown and Korringa[11] point out that, although these three bulk moduli have simple physical interpretations, this "does not necessarily help in knowing their values." Actually the constant K is just the dry frame bulk modulus and has been studied extensively. However, the values of the remaining constants K_s and K_ϕ are not known unless the porous frame is homogeneous on the microscopic scale in which case $K_s = K_\phi = K_m$, the bulk modulus of the constituent material. One of the main results of this paper will be formulas which provide estimates of K_s and K_ϕ in terms of constituent grain moduli when the porous frame is *not* microscopically homogeneous.

In Section II, Biot's equations of poroelasticity are presented. The solution of the single-scattering problem for a spherical inhomogeneity is discussed in Section III and then used in the formulation of a self-consistent effective medium approximation in Section IV. The results are compared with general formulas of Brown and Korringa[11] in Section V. We find that the effective medium approximation gives explicit formulas for the coefficients which are completely consistent with the results of Gassmann,[13] of Biot and Willis,[14] and of Brown and Korringa.[11]

II. Equations of Poroelasticity

Consider two porous media (i.e., host and inclusion) each of whose connected pore space is saturated with a single-phase viscous fluid. The fraction of the total volume occupied by the fluid is the void volume fraction or porosity ϕ, which is assumed to be uniform within each porous constituent but which may vary between the the host and inclusion. The bulk modulus and density of the fluid are K_f and ρ_f, respectively, in the host. The bulk and shear moduli of the (dry) porous frame for the host are K and μ. Parameters for the inclusion will be distinguished by adding a prime superscript. As usual we assume the frame *of the inclusion* is composed of a single constituent whose bulk and shear moduli and density are K'_m, μ'_m, and ρ'_m; however, we need to make no such assumption about the host. The frame moduli may be measured directly or they may be estimated using one of the many methods developed to estimate elastic constants of composites.

For long-wavelength disturbances ($\lambda > h$, where h is a typical pore size) propagating through such a porous medium, we define average values of the (local) displacements in the solid and also in the saturating fluid. The average displacement vector for the solid frame is $\vec{u}$ while that for the pore fluids is $\vec{u}_f$. The average displacement of the fluid relative to the frame is $\vec{w} = \phi(\vec{u}_f - \vec{u})$. For small strains, the frame dilatation is

$$e = e_x + e_y + e_z = \vec{\nabla} \cdot \vec{u}, \tag{8}$$

where e_x, e_y, e_z are the Cartesian strain components. Similarly, the average fluid dilatation is

$$e_f = \vec{\nabla} \cdot \vec{u}_f \tag{9}$$

(e_f also includes flow terms as well as dilatation) and the increment of fluid content is defined by

$$\varsigma = \vec{\nabla} \cdot \vec{w} = \phi(e - e_f). \tag{10}$$

With these definitions, Biot[1-3] shows that the strain-energy functional for an isotropic, linear medium is a quadratic function of the strain invariants[15] $I_1 = e, I_2$, and of ς having the form

$$2E = He^2 - 2Ce\varsigma + M\varsigma^2 - 4\mu I_2, \tag{11}$$

where

$$I_2 = e_y e_z + e_z e_x + e_x e_y - \frac{1}{4}(\gamma_x^2 + \gamma_y^2 + \gamma_z^2), \tag{12}$$

and $\gamma_x, \gamma_y, \gamma_z$ are the shear strain components.

With time dependence of the form $exp(-i\omega t)$, the Fourier transformed version of the coupled wave equations of poroelasticity in the presence of dissipation take the form

$$\mu\vec{\nabla}^2\vec{u} + (H - \mu)\vec{\nabla}e - C\vec{\nabla}\varsigma + \omega^2(\rho\vec{u} + \rho_f\vec{w}) = 0, \tag{13}$$

$$C\vec{\nabla}e - M\vec{\nabla}\varsigma + \omega^2(\rho_f\vec{u} + q\vec{w}) = 0, \tag{14}$$

where

$$\rho = \phi\rho_f + (1-\phi)\rho_m \tag{15}$$

and

$$q = \rho_f[\alpha/\phi + iF(\xi)\eta/\kappa\omega]. \tag{16}$$

The kinematic viscosity of the liquid is η, the permeability of the porous frame is κ, and the dynamic viscosity factor is given (for our present choice of sign for the frequency dependence) by

$$F(\xi) = \frac{1}{4}\xi T(\xi)/[1 + 2T(\xi)/i\xi], \tag{16}$$

where

$$T(\xi) = \frac{ber'(\xi) - ibei'(\xi)}{ber(\xi) - ibei(\xi)} \tag{17}$$

and

$$\xi = (\omega h^2/\eta)^{\frac{1}{2}}. \tag{18}$$

The functions $ber(\xi)$ and $bei(\xi)$ are the real and imaginary parts of the Kelvin function. The dynamic parameter h is a characteristic length generally associated with (and comparable in magnitude to) the steady-flow hydraulic radius. The tortuosity $\alpha \geq 1$ is a pure number related to the frame inertia which has been measured recently[8] for porous glass bead samples and has also been estimated theoretically.[16,17]

For the microscopically homogeneous inclusions being considered, the coefficients H', C', and M' are given[14,18,19] by Eqs.(1)-(4) with

$$K'_s = K'_\phi = K'_m. \tag{19}$$

To decouple the wave equations (13) and (14) into Helmholtz equations for three modes of propagation, we note that the displacements $\vec{u}$ and $\vec{w}$ can be decomposed as

$$\vec{u} = \vec{\nabla}\Upsilon + \vec{\nabla}\times\vec{\beta},\ \ \vec{w} = \vec{\nabla}\psi + \vec{\nabla}\times\vec{\chi}, \tag{20}$$

where Υ, are scalar potentials and $\vec{\beta}$,$\vec{\chi}$ are vector potentials. Substituting (20) into Biot's equations (13) and (14), we find they are satisfied if two pairs of equations hold:

$$(\vec{\nabla}^2 + k_s^2)\vec{\beta} = 0,\ \ \chi = -\Gamma_s\vec{\beta}, \tag{21}$$

where $\Gamma_s = \rho_f/q$ and

$$(\vec{\nabla}^2 + k_\pm^2)A_\pm = 0. \tag{22}$$

In this notation, the subscripts +,–, and s refer respectively to the fast and slow compressional waves and the shear wave. The wave vectors in (21) and (22) are defined by

$$k_s^2 = \omega^2(\rho - \rho_f\Gamma_s)\mu \tag{23}$$

and

$$k_\pm^2 = (\omega^2/2\Delta)(b + f \mp [(b-f)^2 + 4cd]^{\frac{1}{2}}), \tag{24}$$

where

$$b = \rho M - \rho_f C,\ \ c = \rho_f M - qC,\ \ d = \rho_f H - \rho C,\ \ f = qH - \rho_f C, \tag{25}$$

with

$$\Delta = MH - C^2. \tag{26}$$

The linear combination of scalar potentials has been chosen to be

$$A_\pm = \Gamma_\pm \Upsilon + \psi, \tag{27}$$

where

$$\Gamma_\pm = d/[(k_\pm\Delta/\omega^2)^2 - b] = [(k_\pm\Delta/\omega^2)^2 - f]/c. \tag{28}$$

With the identification (28), the decoupling is complete.

Since (21) and (22) are valid for any choice of coordinate system, they may be applied to boundary value problems with arbitrary symmetry. Biot's theory has therefore been applied to the scattering of elastic waves from a spherical inhomogeneity in Ref. 20. The results of that calculation will be summarized in the next section.

III. Scattering from a Spherical Inhomogeneity

The full analysis of scattering from a spherical inhomogeneity in a fluid-saturated porous medium is quite tedious. Fortunately, this work has already been done[20] and we may therefore merely quote the relevant results here.

Let the spherical inhomogeneity have radius a. For the moment, we will place no restrictions on the properties of the inhomogeneous region. Thus the frame bulk and shear moduli, the grain bulk modulus, the density, the porosity, and the permeability of a solid inclusion may all be different from those of the host. Furthermore, the bulk modulus, density, and viscosity of the fluid in an inhomogeneous region may also all be different from those of the host fluid. Suppose now that a plane fast compressional wave is generated at a free surface far from the inclusion. Then, if the incident fast compressional wave has the form

$$\vec{u} = \hat{z}\frac{A_0}{ik_+} exp\, i(k_+ z - \omega t), \tag{29}$$

the radial component of the scattered compressional wave contains both fast and slow parts in the far field and is given by

$$\begin{aligned} u_{1r} = {} & (ik_+)^{-1} exp\, i(k_+ r - \omega t)/k_+ r[B_0^{(+)} - B_1^{(+)} \cos\theta - B_2^{(+)}(3\cos 2\theta + 1)/4] \\ & - (ik_-)^{-1} exp\, i(k_- r - \omega t)/k_- r[B_0^{(-)} - B_1^{(-)} \cos\theta - B_2^{(-)}(3\cos 2\theta + 1)/4]. \end{aligned} \tag{30}$$

Then, with the definitions $\kappa_\pm = k_\pm a$ and $\kappa_s = k_s a$ and with no restrictions on the materials, we find that

$$\begin{aligned} B_0^{(-)} = {} & \frac{i\kappa_-^3 A_0}{3M'(\Gamma_+ - \Gamma_-)(K' + \frac{4}{3}\mu)}\Big[(C - M\Gamma_-)(K' + \frac{4}{3}\mu) \\ & - (C' - M'\Gamma_-)(K + \frac{4}{3}\mu) + (C - M\Gamma_-)(C' - M'\Gamma_-)\Big(\frac{C'}{M'} - \frac{C}{M}\Big)\Big], \end{aligned} \tag{31}$$

and

$$B_0^{(+)} = \frac{\kappa_+^3 A_0}{3i}\frac{[K' - K + (C - M\Gamma_-)(C'/M' - C/M)]}{K' + \frac{4}{3}\mu} + (\kappa_+/\kappa_-)^3 B_0^{(-)}. \tag{32}$$

Expansions of the other coefficients in the small parameter $\epsilon = C/K$ have been given in the reference.[20] However, for the present application, only the first two coefficients are needed and these happen to be the only ones known exactly at present. Of course, the full scattered wave also contains transverse components of the compressional wave, relative fluid/solid displacement, and mode converted shear waves. However, the scattering coefficients for these contributions are linearly dependent on the the coefficients in (30) and therefore contain no new information. It is sufficient then to base our discussion on the expression (30).

As an elementary check on our analysis, we should first consider the limit in which the porosity ϕ vanishes. Then the fluid effects disappear from the equations and only the first line of (30) survives. Furthermore, it is not difficult to check[20] that the coefficients $B_n^{(+)}$ for $n = 0, 1, 2$ reduce to the well-known results for scattering from a spherical elastic inclusion in an infinite elastic medium.[21] For example,

$$B_0^{(+)} = -i\kappa_+^3 A_0 (K' - K)/(3K' + 4\mu) \tag{33}$$

in this limit as expected.

IV. Microscopic Heterogeneity

The equations of poroelasticity presented in Section II have several limitations. For example, these equations were derived with an explicit long-wavelength (low-frequency) assumption and also with strong implicit assumptions of homogeneity and isotropy on the macroscopic scale. Another restriction assumes that the pore fluid is uniform and that it fully saturates the pore space. For the present application, we will assume that a single fluid saturates all the pore space for host as well as inclusion and the scattering is caused by microscopic heterogeneity in the solid properties.

Before deriving our main results, consider the problem of the porous frame without a saturating fluid (or with a highly compressible saturating gas). Then the second line of Eq.(30) disappears and only the fast wave terms contribute to the scattering. This limit is formally equivalent to the problem of elastic wave scattering from a spherical inclusion which has been treated in detail previously (see *Ref.* 21 and other references therein). The effective medium approximation requires the weighted average of the single-scattering results to vanish. This approach simulates the physical requirement that the forward scattering should vanish at infinity if the impedance of the "effective medium" has been well matched to that of the composite. The resulting condition is that the volume weighted average of each of the $B_n^{(+)}$'s for $n = 0 - 2$ must vanish. Using the convention that the effective constants for the composite porous medium are distinguished by an asterisk, the formulas for the effective bulk (K^*) and shear(μ^*) moduli for the dry porous frame of a microscopically heterogeneous medium are

$$\frac{1}{K^* + \frac{4}{3}\mu^*} = \left\langle \frac{1}{K(\vec{x}) + \frac{4}{3}\mu^*} \right\rangle \tag{34}$$

and

$$\frac{1}{\mu^* + F^*} = \left\langle \frac{1}{\mu(\vec{x}) + F^*} \right\rangle \tag{35}$$

where

$$F = (\mu/6)(9K + 8\mu)/(K + 2\mu). \tag{36}$$

The spatial($\vec{x}$) average is denoted by $\langle \cdot \rangle$. The remaining constant to be determined is the corresponding effective density which is just given by the average density.[21] For example, Eq.(34) follows easily from the volume average of (33). Note that the equations for K^* and μ^* are coupled and therefore must be solved iteratively (i.e., self-consistently). Although the form of the equations (34) and (35) is identical to that obtained for elastic composites, it is important to recognize that the results can be quite different since the local constants $K(\vec{x})$ and $\mu(\vec{x})$ appearing in the formulas are frame moduli of the constituent spheres of dry porous material, not (necessarily) the moduli of the individual material grains. Of course, since the formula reduces correctly in the absence of porosity to the corresponding result for the purely elastic limit, the user of Eqs.(34) and (35) has some discretion about conceptually lumping grains together to form a porous frame or treating them as isolated elastic inclusions. For purposes of modelling complex aggregates of grains typical of earth materials, this freedom of choice appears to be a real advantage.

Now we will restrict discussion to the very low frequency limit where

$$\Gamma_+ = H/C \tag{37}$$

and

$$\Gamma_- = 0. \tag{38}$$

With these restrictions, the relevant scattering coefficients reduce to

$$\begin{aligned} B_0^{(-)} = \frac{i\kappa_-^3 C A_0}{3HM'(K' + \frac{4}{3}\mu)} \Big[& C(K' + \frac{4}{3}\mu + \sigma' C') \\ & - C'(K + \frac{4}{3}\mu + \sigma C) \Big], \end{aligned} \tag{39}$$

and

$$B_0^{(+)} = \frac{\kappa_+^3 A_0}{3i} \frac{[K' - K + (\sigma' - \sigma)C]}{K' + \frac{4}{3}\mu} + (\kappa_+/\kappa_-)^3 B_0^{(-)}. \tag{40}$$

The resulting conditions on the effective constants are

$$\left\langle \frac{C^*(K(\vec{x}) + \frac{4}{3}\mu^* + \sigma(\vec{x})C(\vec{x})) - C(\vec{x})(K^* + \frac{4}{3}\mu^* + \sigma^* C^*)}{M(\vec{x})(K(\vec{x}) + \frac{4}{3}\mu^*)} \right\rangle = 0 \tag{41}$$

and

$$\left\langle \frac{K(\vec{x}) - K^* + (\sigma(\vec{x}) - \sigma^*)C^*}{K(\vec{x}) + \frac{4}{3}\mu^*} \right\rangle = 0. \tag{42}$$

Note that (41) and (42) depend on the effective medium frame moduli K^* and μ^* determined by (34) and (35). The new constants which are determined by (41) and (42) are C^* and σ^*. The expressions for C^* and σ^* are coupled as written but may be uncoupled after some algebra. The final expressions for these constants are

$$C^* = \sigma^* \Big/ \left[\left\langle \frac{1}{M(\vec{x})} \right\rangle + \left\langle \frac{\sigma^2(\vec{x}) - (\sigma^*)^2}{K(\vec{x}) + \frac{4}{3}\mu^*} \right\rangle \right] \tag{43}$$

and

$$\sigma^* = \left\langle \frac{\sigma(\vec{x})}{K(\vec{x}) + \frac{4}{3}\mu^*} \right\rangle \Big/ \left\langle \frac{1}{K(\vec{x}) + \frac{4}{3}\mu^*} \right\rangle. \tag{44}$$

Notice that (44) does not depend on C^*; therefore, both constants have values determined explicitly by the formulas. In contrast, the frame moduli K^* and μ^* are determined only implicitly by (34) and (35).

V. Consistency

To check the consistency of the results of the previous section, the forms of (43) and (44) must be compared to those of (2) and (4) as derived by Brown and Korringa.[11] The key point to check is that the forms of (43) and (44) allow the resulting effective bulk moduli K_s^* and K_ϕ^* to be independent of the properties of the saturating fluid. Rearranging (4) for the effective medium, we find that

$$K_s^* = K^*/(1-\sigma^*) \tag{45}$$

which clearly is independent of the fluid properties since $\sigma(\vec{x})$ and all the material moduli appearing in (44) are independent of the fluid. Equating (43) and (2), we find

$$\phi^*/K_\phi^* = \sigma^*(1-\sigma^*)/K^* + \phi^*/K_f - \Big[\Big\langle \frac{1}{M(\vec{x})} \Big\rangle + \Big\langle \frac{\sigma^2(\vec{x}) - (\sigma^*)^2}{K(\vec{x}) + \frac{4}{3}\mu^*} \Big\rangle\Big]. \tag{46}$$

Then, using (2) and (3), it is easy to see that the terms depending on K_f in (46) all cancel. The general form of this modulus is given by

$$\phi^*/K_\phi^* = \sigma^*(1-\sigma^*)/K^* - \Big[\Big\langle \frac{\sigma(\vec{x}) - \phi(\vec{x})}{K_m(\vec{x})} \Big\rangle + \Big\langle \frac{\sigma^2(\vec{x}) - (\sigma^*)^2}{K(\vec{x}) + \frac{4}{3}\mu^*} \Big\rangle\Big]. \tag{47}$$

When the porous material is homogeneous on the microscopic scale, it is not difficult to show that (45) and (47) reduce to the expected identities

$$K_s^* = K_\phi^* = K_m. \tag{48}$$

To check the predictions of this approximation, consider a porous medium composed of two types of solids constituents with constants given by $K_m^{(1)} = 7.0\ GPa$, $\mu_m^{(1)} = 5.0\ GPa$, $\phi^{(1)} = 0.1$, $K^{(1)} = 5.4\ GPa$, $\mu^{(1)} = 3.9\ GPa$ for the softer constituent and by $K_m^{(2)} = 41.0\ GPa$, $\mu_m^{(2)} = 30.0\ GPa$, $\phi^{(2)} = 0.3$, $K^{(2)} = 14.9\ GPa$, $\mu^{(2)} = 10.9\ GPa$ for the harder constituent. The effective frame moduli are first calculated by iteration using (34) and (35). Then the result for σ^* is found using (44). Finally, (45) and (47) are used to compute K_ϕ^* and K_s^* and then the results for these bulk moduli are plotted in Figure 1. We see that the two moduli vary smoothly from one limiting value $K_m^{(1)}$ to the other $K_m^{(2)}$ as the relative volume fraction of the two porous constituents varies from zero to one. Note that $K_\phi^* > K_s^*$ for all values of volume fraction except the end points where (48) applies. The observed variation is similar to that found in various rigorous bounds on elastic constants of composites but there is no reason to believe this observed similarity is more than coincidence. Notice that no pore fluid need be specified in this calculation until we want to calculate the coefficient C^* for use in Biot's equations.

VI. Conclusions

For porous materials with more than one variety of solid component, a self-consistent effective medium approximation has been constructed which gives explicit estimates of the coefficients in Biot's equations of poroelasticity. It has been shown that these formulas are completely consistent with the known general results for these coefficients. In particular, the estimates obtained for the moduli K_s and K_ϕ are always independent of K_f and also reduce correctly to K_m in the trivial limit of microscopic homogeneity. These results are, of course, not exact but should give good estimates of these coefficients when the assumption that the homogeneous porous components are spherically shaped is at least approximately satisfied. The method used here can be generalized to other shapes of the porous components but to do so requires the explicit solution of the scattering problem for the shapes of interest. Some progress on this more difficult problem has been made[22] but we will not pursue this line of thought further at the present time.

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Figure Caption

Figure 1. Comparison of the computed values of the solid bulk moduli K_ϕ^* and K_s^* for a porous medium composed of two types of solids constituents as the relative volume fraction of the two porous constituents varies. The relevant constants are given by $K_m^{(1)} = 7.0\ GPa$, $\mu_m^{(1)} = 5.0\ GPa$, $\phi^{(1)} = 0.1$, $K^{(1)} = 5.4\ GPa$, $\mu^{(1)} = 3.9\ GPa$ for the softer constituent and by $K_m^{(2)} = 41.0\ GPa$, $\mu_m^{(2)} = 30.0\ GPa$, $\phi^{(2)} = 0.3$, $K^{(2)} = 14.9\ GPa$, $\mu^{(2)} = 10.9\ GPa$ for the harder constituent. The effective frame moduli are first calculated by iteration using (34) and (35). Then result for σ^* is found using (44). Finally, (45) and (47) are used to compute the curves.

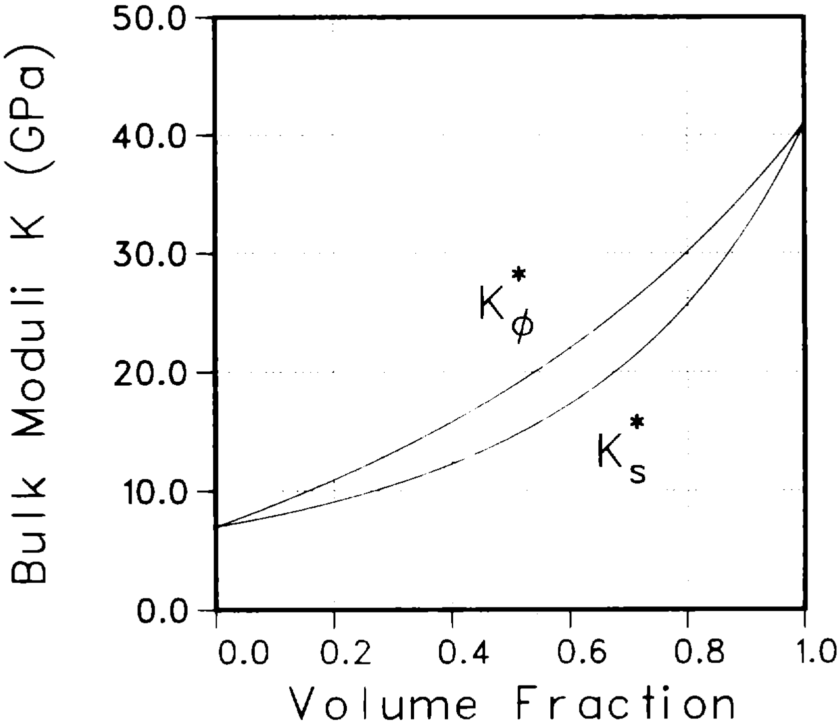
Bulk Moduli K (GPa)
50.0
40.0
30.0
20.0
10.0
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Volume Fraction
K^*_ϕ
K^*_s

FIGURE 1